QUASILINEAR METHOD FOR A JOINT DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS OF MATERIALS

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A method is described for jointly determining the thermophysical characteristics of materials. The method exploits certain features in the development of the initial stage of a regular regime.

The initial stage of the regular regime for a hollow cylinder was studied in [1]. Because of certain characteristic features of this stage, it can be defined as the quasilinear heating regime. A corresponding method for jointly determining the thermophysical characteristics permits experiments to be carried out with a single basic thermocouple based on the outside surface of the sample, without disrupting the geometrical arrangement. A "single-point" measurement method was proposed in [2] for the case of adiabatic heating and, independently, in [3], in connection with quasisteady methods.

In the quasilinear regime, the time dependence of the temperature for a hollow cylinder heated at its inner surface by a source of constant strength under the condition of natural heat transfer with a medium at a constant temperature is described by [1]

$$\theta = D + A_1 \mathrm{Fo}, \tag{1}$$

where

$$\theta = \frac{\Delta T}{Q_1}\lambda, \quad \Delta T = T - T_0, \tag{2}$$

$$D = \frac{1}{2\pi\,\mathrm{Bi}}(1 - \mathrm{Bi}\ln\rho) - D_1, \quad A_1 = D_1 s_1^2, \tag{3}$$

$$D_1 = \frac{J_1(s_1\rho_0)\,Y_0(s_1\rho) - Y_1(s_1\rho_0)\,J_0(s_1\rho)}{2\rho_0 s_1\left[(\mathrm{Bi}^2 + s_1^2)\,\Phi_1^2 - 1\right]},$$

$$\Phi_1 = \frac{J_1(s_1\rho_0)}{s_1 J_1(s_1) - \mathrm{Bi}\,J_0(s_1)}.$$

The number s_1 is determined as the first root of the characteristic equation

$$\mathrm{Bi}\left[J_1(s\rho_0)\,Y_0(s) - Y_1(s\rho_0)\,J_0(s)\right] - s\left[J_1(s\rho_0)\,Y_1(s) - Y_1(s\rho_0)\,J_1(s)\right] = 0.$$

Dependence (1) can be used for a joint determination of the thermophysical characteristics of materials. Transforming to dimensional quantities and differentiating (1) with respect to the time, we find equations for a, λ and c:

$$a = \frac{DR^2}{A_1\Delta}\cdot\frac{dT}{dt}, \tag{4}$$

$$\lambda = \frac{Q_1 D}{\Delta}, \tag{5}$$

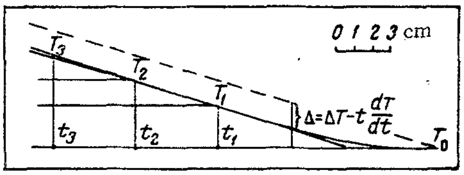

Fig. 1. Illustrative oscilloscope traces showing the recorded temperature for polymethyl methacrylate.

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TABLE 1. Values of the Quantities $-D \cdot 10^2$ (I) and $A_1 \cdot 10$ (II)

Bi	ρ_0					Bi	ρ_0				
	0,1	0,2	0,4	0,6	0,8		0,1	0,2	0,4	0,6	0,8
			I						*II*		
0,005	3,68	3,19	2,19	1,31	0,59	0,005	3,215	3,315	3,788	4,972	8,841
0,010	3,68	3,19	2,18	1,31	0,59	0,010	3,215	3,315	3,787	4,971	8,839
0,025	3,66	3,18	2,18	1,30	0,59	0,025	3,214	3,313	3,784	4,968	8,835
0,050	3,65	3,17	2,18	1,30	0,59	0,050	3,213	3,310	3,780	4,961	8,829
0,075	3,64	3,16	2,16	1,30	0,59	0,075	3,211	3,307	3,775	9,955	8,822
0,100	3,63	3,14	2,15	1,29	0,58	0,100	3,210	3,303	3,769	4,949	8,816
0,250	3,52	3,06	2.10	1,27	0,58	0,250	3,194	3,281	3,736	4,911	8, 776

$$c = \frac{A_1 Q_1}{\gamma R^2 \dfrac{dT}{dt}}, \tag{6}$$

$$\Delta = \Delta T - t\frac{dT}{dt}. \tag{7}$$

In Eqs. (4)-(6), the quantities D and A_1 are calculated theoretically, while Δ and dT/dt are found from the quasilinear part of the time dependence of the temperature at a certain time t. The thermocouple which monitors the temperature is mounted on the outer surface at the middle of the sample.

The quantities D and A_1 which appear in Eqs. (4)-(6) depend on the Biot number and the dimensionless inner radius ρ_0. The values of these quantities calculated for the outer surface ($\rho = 1$) are listed in Table 1, from which we see that for the values of the Biot number under consideration the change in D does not exceed 5%, while that in ~4.5% does not exceed A_1 ~ 1.5% for all ρ_0.

On the basis of the solution in [1] of the problem of the temperature field in a cylinder, with heat transfer takes into account, an equation was derived for evaluating the Biot number from the same experimental data used to determine the thermophysical characteristics. For Bi < 0.25 this equation is

$$\mathrm{Bi} = \frac{\Delta}{k(t_2 - t_1)\dfrac{dT}{dt}} \ln\frac{T_2 - T_1}{T_3 - T_2}. \tag{8}$$

Here Δ and dT/dt are determined from the linear part of the time dependence of the temperature (see the discussion above). The temperatures T_1, T_2, and T_3 are determined for times t_1, t_2, and t_3 under the condition $t_2 - t_1 = t_3 - t_2$ in the region adjacent to the quasilinear region (Fig. 1). The coefficient k depends on the dimensionless inner radius ρ_0 (for ρ_0 = 0.1; 0.2; 0.4; 0.6; and 0.8, the values of k are 0.234, 0.201, 0.137, 0.082, and 0.037).

The analysis of Eqs. (4)-(5) is based on the quasilinear region, whose extent increases with decreasing Biot number [1]. In the proposed method for determining the thermophysical characteristics, small values of the Biot number are achieved by using small samples and low heating rates. Under these conditions, the temperature drops over the cross section of the sample are also small (ΔT < 5°), so that it becomes possible to average the thermophysical characteristics without introducing any important errors.

The measurement apparatus consists of standard equipment. To determine the temperature dependence of the thermophysical characteristics of the materials, we use a muffle oven with a working chamber 20 mm in diameter and 130 mm long, designed for heating to 1200°C. The sample, with an outer diameter of 10 mm, an inner diameter of 2-3 mm, and a length of 50 mm, is placed in the central part of the working chamber. In these experiments, the sample is simply suspended from a manganese heating filament (0.1 mm in diameter), so that this filament is under tension. The filament is arranged along the axis of the sample by means of centering grooves. The working chamber is tightly covered with ceramic caps with leads for the thermocouple and for the power supply of the heating filament. The temperature is measured with a copper–Constantan (or Chromel–Copel) thermocouple (0.09-0.12 mm). Both junctions of the thermocouple are within the working chamber; one is at the center of the outer surface of the sample, while the second is far from it. Since the heating power is low, the second junction remains at essentially its initial temperature during the experiment (< 60 sec). The heating filament is supplied a regulated voltage from a type VS-26 dc power supply. The power is determined with a type M1107 ammeter and

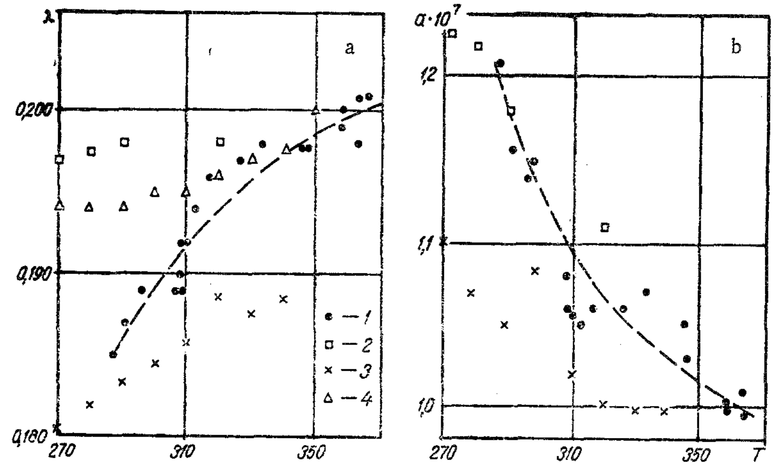

Fig. 2. Thermal conductivity (a) and thermal diffusivity (b) as functions of the temperature for polymethyl methacrylate. 1) According to the present data; 2) according to [5] for $\gamma = 1174$ kg/m^3; 3) according to [5] for 1194 kg/m^3; 4) according to [4].

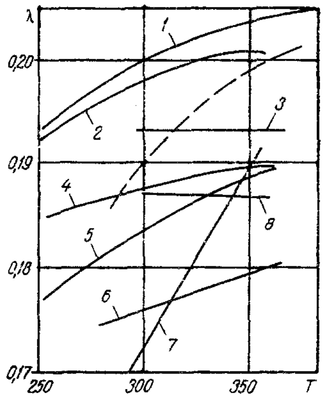

Fig. 3. Thermal conductivity of polymethyl methacrylate as a function of the temperature according to data from various studies. 1) [6]; 2) [4]; 3) [7]; 4) [8]; 5) [10]; 6) [11]; 7) [9]; 8) [12]. The dashed curve shows the data from the present study. Here λ is in W/m · deg^{-1} and T is in °K.

a type M1109 voltmeter. Triggered simultaneously with the power supply is a type N-700 oscilloscope, through a type F359 "photocompensation" amplifier. The temperature is recorded on light-sensitive paper. A pulse marks the beginning of a recording. This apparatus is used for measurements with a variety of materials, both conductors and dielectrics.

To test the method, we carried out measurements on materials recommended as standard materials [4]: polymethyl methacrylate and Armco iron. Figure 1 shows an illustrative oscilloscope trace for polymethyl methacrylate at an initial temperature of $T_0 = 371$°K. This recording was carried out at a chart speed of 2.5 mm/sec. The temperature scale is 0.05 deg/mm. The sample was a cylinder 50 mm long with an outer diameter of 10 mm and an inner diameter of 3 mm. The heater power was 5.7 W /m. From the oscilloscope trace in Fig. 1 we find $Bi \approx 0.2$, in agreement with Eq. (8). The middle of the quasilinear region ($t_1 = 51.8$ sec) corresponds to a surface temperature $\Delta T = T_1 - T_0 = 1.2°$. The calculated temperature drop between the outer and inner surfaces is ~5.0°.

Figure 2a shows measurements of the thermal conductivity λ for polymethyl methacrylate. The points are plotted on the basis of measurements made by the method described above. The dashed curve shows the function λ(T) which is established. Shown for comparison are values of the thermal conductivity according to measurements by other investigators. For the sample of the present experiments we have $\gamma \approx 1174$ kg/m^3. Figure 2b shows data on the thermal diffusivity a. The discrepancy between the results does not exceed 6% over the temperature range considered, in agreement with a theoretical estimate of the accuracy of the method.

The samples used in [5] were solid cylinders $l = 10$-140 mm long and d = 30-40 mm in diameter or composite tubular samples 100 mm long with an outer diameter of d = 50 mm and an inner diameter of $d_0 = 30$ mm. The samples used in [4] were $l = 120$ mm long with an inner diameter of $d_0 = 4$ mm and an outer diameter of 40 mm.

There have been many studies of polymethyl methacrylate. Figure 3 shows the temperature dependence of the thermal conductivity according to the data from the various studies [4]. The dashed curve corresponds to the present experiments. Analysis of Figs. 2 and 3 reveals that the results found by the present method agree quite well with the results found by other investigators.

The present measurements for Armco iron at temperatures up to ~1100°K also agree quite well with earlier results.

In view of the simplicity of the present method, we can hope that it will find laboratory applications.

NOTATION

T	is the cylinder temperature;
T_0	is the initial temperature of cylinder and ambient medium;
$\rho = r/R$	is the dimensionless radial coordinate;
$\rho_0 = R_0/R$	is the dimensionless inner radius;
R	is the outer radius of cylinder;
Q_1	is the power per unit length supplied to the cylinder;
$Fo = (a/R^2)t$	is the Fourier number;
t	is the time;
$Bi = (\alpha/\lambda)R$	is the Biot number;
$J_0(s\rho)$, $J_1(s\rho)$	are the Bessel functions of the first kind;
$Y_0(s\rho)$, $Y_1(s\rho)$	are the Bessel functions of the second kind;
a	is the thermal diffusivity;
λ	is the thermal conductivity;
α	is the heat-transfer coefficient;
γ	is the density of material.

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